**Gardner’s relations and AVO inversion**

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**Introduction**

Seismic reflection data contain information about the nature and the composition of a medium (its lithology, fluid content, porosity and so on). The lithological and fluid properties of a medium cannot be inferred from P-wave data alone: information on the S-wave response is also required. Although S-wave data are rarely directly acquired, S-wave information can be extracted from the P-wave amplitude-variation-with-offset (AVO). One consequence of this is a spurious correlation between estimated P-wave and S-wave properties; another is that S-wave related estimates are very sensitive to noise (Cambois 2000).

AVO inversion is commonly applied as a means of retrieving P-wave and S-wave properties from seismic data. The use of a constraint, such as a Gardner-type relation, is highly recommended because, by reducing the number of parameters, it stabilizes the inversion. But the constraint introduces biases into the estimated parameters. As our ultimate aim is quantitative (rather than only qualitative) characterization of fluid-filled reservoirs, we studied the quantitative effects of using a Gardner constraint on AVO inversion. This paper shows results from synthetic models based on log data from four (one per AVO class) North Sea wells. A real data example from the North Sea illustrates the application of these results.

**AVO model and AVO inversion**

The Zoeppritz equation for the P-wave reflection coefficient $R_p(\theta)$ expresses the variation of the amplitude of a plane P-wave reflected from a single interface as a function of the incidence angle $\theta$ and six independent elastic parameters (three on each side of the reflecting interface). However, this equation is rather complex and sheds little insight into the relative importance and effect of each parameter. Aki & Richards (1980) assumed small contrasts in elastic properties at the interface to linearize the Zoeppritz equation at pre-critical incidence angles:

$$R_p(\theta) = R_p(0) + G\sin^2\theta + C\sin^2\theta\tan^2\theta. \quad (1)$$

The reflection parameters (the intercept $R_p(0)$, gradient $G$ and curvature $C$) are combinations of the P-wave and S-wave properties and densities of the two media.

AVO inversion consists of retrieving medium parameters (e.g. contrasts and ratios of P-wave, S-wave impedances and contrasts of densities) from the seismic reflection data. However even in a moderately noisy data case, inversion for three parameters per layer is very unstable. One needs to introduce a constraint on the parameters, such as a Gardner-type relation, to stabilize the inversion of seismic data (Wang 1999).

**Gardner-type relation**

Although the bulk density $\rho$ of any rock may be easily related to the matrix or grain density $\rho_m$, the pore fluid density $\rho_f$ and its porosity $\Phi$:

$$\Phi = (1 - \theta)\rho_m + \theta\rho_f, \quad (2)$$

the velocity is often not simply related to the porosity, therefore to the density. This is mostly due to the presence of microcracks, which strongly affect the P-wave and S-wave velocities (Mavko et al. 1998). Assuming high effective pressures and fluid saturation, both of which minimize the effect of cracks, Gardner et al. (1974) established, for a wide range of velocity and porosity values, a relation between the P-wave velocity $V_p$ and the density $\rho$. In terms of contrasts this relation yields:

$$\frac{\Delta \rho}{\rho} = \frac{n}{V_p}, \quad (3)$$

Using such an equation, even with $n$ specific to each rock as suggested by Castagna et al. (1993), ignores the effect of several factors such as lithological or fluid changes: Gardner’s relations mainly mimic changes in porosity. Indeed, Fig. 1a shows velocity and density values from four wells (one per AVO class) in the North Sea for brine-filled sandstone (open circles) and shale seals (asterisks). Superimposed are the density-velocity values predicted by the Gardner relations of Castagna et al. (1993) for sandstone (solid line) and shale (dashed line). Except in the class III case, no density change at the interface separating shale from sandstone follows a Gardner-type relation. Actually the change is more or less perpendicular to the predicted curves: Gardner’s prediction fails
in this practical example. Moreover the variations from fluid changes predicted by Gassmann (1951)–Biot (1962) theory do not obey Gardner’s relation. The variations of density with P-wave velocity for a gas-fill (green dashed line), an oil-fill (red dot-dashed line) and a brine-fill (blue solid line) class III sandstone are shown on Fig. 1b. The plus signs denote 30% porosity, the asterisks 35% porosity. For the fluid substitution, the (matrix) grain was assumed to be pure quartz, its density to be 2.65 g/cm³ and its bulk modulus 36 Gpa. The P-wave velocities and densities used to characterize the fluids are gathered in Table 1. I also assumed a P-wave velocity to S-wave velocity ratio to be 1.5 for the dry frame. In this example again, a Gardner-type relation fails: although it may predict well a porosity change within a given fluid-filled rock, it cannot predict the density change due to a fluid substitution at fixed porosity.

However, as previously mentioned, the use of a constraint is required to stabilize AVO inversion. Figure 2 shows intercept, gradient and curvature estimates from noisy data synthesized for a single interface separating shale from a class IV brine-filled (blue crosses) or gas-filled (green open circles) sandstone. The gradient values are plotted against the intercept values (top row), the curvature against the gradient values (second row), in the cases of a 3-term unconstrained inversion (left column) and of a 3-term Gardner-constrained inversion (right column). The use of the constraint yields less noise-prone gradient and curvature estimates. In the gas-fill case, for instance, the mean-squared error (MSE) for the gradient exceeds 20 after an unconstrained inversion. But when retrieved by constrained inversion the MSE in the gradient is less than 1. The distinction between fluids is also visually improved. Note that, because of the Gardner constraint (Eqn 3), the curvature estimates in Fig. 2 bottom right are just a scaled 

\[ \frac{1}{n+1} \] 

version of the intercept estimates.

To attempt quantitative AVO inversion, we need to estimate the biases induced by Gardner’s relation.

**Reservoir synthetic models and inversion strategies**

Four single interface models were set up using densities, P-wave and S-wave velocities well logs from four North Sea reservoirs (one per AVO class). The parameters of the models (Table 2) were obtained by averaging log values on clean sandstone intervals (3–10 m thick) for the sandstone reservoir and within the shale seal (with slightly larger intervals than for sandstones) for the shale parameters. The S-wave velocities of the class I model were based on laboratory measurements, not logs. The parameters for the gas-filled and oil-filled sandstone reservoirs, when not provided by the logs, were obtained by fluid substitution using Gassmann’s equations.

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**Table 1** Fluid properties of the brine, the oil and the gas

<table>
<thead>
<tr>
<th>Fluid</th>
<th>Density (g/cm³)</th>
<th>P-wave velocity (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brine</td>
<td>1.050</td>
<td>1485</td>
</tr>
<tr>
<td>Gas</td>
<td>0.300</td>
<td>1016</td>
</tr>
<tr>
<td>Oil</td>
<td>0.900</td>
<td>1438</td>
</tr>
</tbody>
</table>

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**Figure 1**

(a) The Gardner velocity-density variations for sandstone (solid line) and shale (dashed line) from Castagna et al. (1993). The velocities and densities of brine-filled sandstones (open circles) and shale seals (asterisks) based on four North Sea reservoirs are also shown.

(b) The velocity-density variations for a class III sandstone reservoir having brine-fill (blue dashed line), oil-fill (red dot-dashed line) and gas-fill (green solid line). The oil-fill and gas-fill curves were predicted by Gassmann fluid substitution from the Gardner-type brine-fill curve. The plus signs denote 30% porosity and the asterisks 35% porosity.
Synthetic-model results

The P-wave and S-wave impedance estimates for the synthetic models from the Gardner-constrained inversion (open circles) are cross-plotted and compared with the model values (asterisks) on Fig. 3. It is the S-wave impedance that is biased by the constraint; the P-wave impedance is not perceptibly affected. In class I and II cases, where the sand-shale change does not follow a Gardner-relation (Fig. 1a) and where fluid effects are weaker, the most bias is shown by the brine-fill S-wave impedance (≈ 2.5% lower than actual model values). In the class III case, where the sand-shale change was almost consistent with a Gardner-relation (Fig. 1a), hardly any bias occurred in the brine-fill case. But when the sand was filled with gas, the bias was much more severe (≈ 8% lower than actual model values). In the class IV case, the biases remained small (less than 1%) for all fluid content.

We studied the effects of other kinds of constraints, such as a soft constraint or by neglecting the density contrast (respectively the first and the second inversion strategies mentioned by Swan 1993). We found that the induced biases depend on the AVO class, on the fluid content and on the constraint itself. All strategies biased the S-wave related estimates, whereas the P-wave related parameters were only very slightly affected.

Real data example

Data from a North Sea field showed three gas caps of a Class III type. A three-term Gardner-constrained inversion of the
trough amplitudes corresponding to the top-reservoir provided an intercept (Fig. 4) and a gradient value maps. We selected two areas: one covering the thicker part of the large gas cap and another down dip from the gas cap where the porefill was brine. The gradient values were plotted against the intercept values for both selected areas (Fig. 5). The direction of the noise trend (light blue line) was computed from the range of incidence-angles (Hendrickson 1999) at the top of the reservoir. A median-filter was applied to the intercept-gradient values along the noise trend to suppress the effect of noise (Fig. 6). The expected values from logs and close well tie (represented as squares on Fig. 6) showed a negative bias in the

Figure 4  AVO intercept map over three gas caps (in blue) in the North Sea based on the amplitudes of the top reservoir reflection. Division by $10^5$ converts amplitudes to approximate reflection coefficients.

Figure 5  AVO intercept and gradient crossplot of the top reservoir reflection from areas 1 (blue) and 2 (red) of Fig. 4, showing the noise trend direction (light blue line).
AVO gradient over the gas cap: this agreed with the synthetic-model results. The trend with slope ≈ 2.8 is that expected from variations in sandstone porosity, while the steeper negative slopes may correspond to variations in shaliness.

**Conclusions**

The use of a Gardner-type relation as a constraint in AVO inversion reduced the noise-induced variance of estimates from AVO inversion but at the same time introduced biases into the S-wave related estimates. These biases depended on the AVO class, on the fluid content and on the constraint itself.

We used noise-free synthetic models to identify the origin of the biases. In the class I and II examples, it was the inability of a Gardner relation to predict the density-velocity change at the sand–shale interface which was mainly responsible for the biases. It appears that the greater the impedance contrast, the larger the bias. In the class III case, the bias originated primarily from the failure of the Gardner relation to predict the effects of a fluid-content change: the S-wave impedance of the gas-fill was strongly biased. In no other case did the bias reach 8%. In the class IV example, the biases did not exceed 1%.

A possible strategy for quantitative AVO inversion would be to correct the inverted seismic data from the constraint-induced biases by quantitatively estimating the biases on noise-free synthetic data set up from log data and inverted using the same constraint.

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**References**


