Directional deconvolution of the seismic source signature combined with prestack migration

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Modern marine seismic source arrays are highly directional. The directivity of the far-field source signature is ignored in conventional one-dimensional (1D) deconvolution. For a single seismic reflection profile it is assumed that all arrivals arise from raypaths lying in the in-line plane, and so some kind of two-dimensional (2D) deconvolution operator is required to remove the directivity of the source signature. This may be achieved in combination with prestack migration.

Conceptually, the prestack migration is done by taking each data sample on a constant-offset section and smearing it along a locus on the migrated section. This locus is an ellipse for constant velocity. There is a one-for-one correspondence between points on the migration ellipse and the take-off angle of the waveform radiated from the source. So the constant-offset section may be deconvolved before migration for the source waveforms radiated in different directions, and the output values distributed at the appropriate points on the migration ellipses. Suitable weighting factors may be chosen for the migration process, and a spectral modification factor should be applied.

Directional deconvolution of the seismic source signature could also be implemented in combination with dip moveout (prestack partial migration of constant-offset sections to zero offset) and could be extended to three-dimensional (3D) data and to incorporate receiver directivity.

We illustrate the technique using 2D data from a physical modelling system.

Introduction

In a uniform medium, only a spherically symmetric seismic source radiates the same waveform in all directions. In the marine case, even a single source element (e.g. an airgun or watergun) a few metres below the surface will be a directional source because of the ghost reflection from the sea surface. Modern marine seismic source arrays for reflection work combine several source elements in an areal array and are highly directional. For watergun arrays, the source signature radiated in any direction may be estimated by superimposing the signatures of individual guns (Tree, Lugg and Brummitt 1986). For airgun arrays there is a large interaction effect (Safar 1976). Nevertheless, by deploying at least one near-field hydrophone for each gun, it is possible to determine the source signature radiated in all directions (Ziolkowski et al. 1982; Parkes et al. 1984).

The directivity of source arrays is a desirable feature because most of the noise on marine seismic records is source-generated, e.g. from sea-water layer multiples or diffractions of water-borne arrivals. These arise from raypaths which leave the source at large take-off angles (i.e. large angles to the vertical) whereas the desired reflected arrivals leave the source at small take-off angles. Directivity in both in-line and cross-line directions is achieved by extending the respective dimensions of the array together with a suitable choice of source elements (e.g. Parkes, Hatton and Haugland 1984). In the following we shall confine discussion to two dimensions, although the argument and proposed method of directional deconvolution could readily be extended to three dimensions.

The disadvantage of having a directional source is that by reducing the amplitude of the source signature at large take-off angles, the source signature varies with direction at all take-off angles. This is because directivity increases with frequency. Consequently, primary reflected arrivals from different horizons have different waveforms as shown schematically in Fig. 1. The seismic wavelet varies down the seismogram according to the take-off angle of each arrival.

An example of the in-line directivity of an airgun array is shown in Fig. 2a, taken from Loveridge et al. (1984). They designed a signature deconvolution filter for the vertically travelling waveform and applied it to the waveforms radiated at all take-off angles, with the disastrous consequences shown in Fig. 2b. Of course, reflections from rays leaving the source at large take-off angles will be muted out. Nevertheless, there is considerable directivity even at small take-off angles.

The crux of the directional deconvolution problem is to deconvolve each arrival with a filter designed for the waveform leaving the source at the right take-off angle, without knowing what that take-off angle actually is. Here we propose and demonstrate a solution which combines 1D deconvolution for source signatures at a full range of take-off angles with prestack migration.

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Fig. 1. Schematic diagram showing seismogram of primary reflections only from horizontal layers. The wavelet changes down the seismogram because of source directivity.

Prestack migration

Prestack migration can be almost completely understood by geometrical considerations. Figure 3a shows a constant-offset section which contains only an isolated arrival. Assuming constant velocity, the reflector configuration must be the depth ellipse shown in Fig. 3b. The equation of the ellipse depends upon the traveltime of the arrival, the velocity and the offset between source and receiver.

Conceptually, migration may be performed by taking the event from the constant-offset section and distributing it around the ellipse. (We shall refer to this ellipse as the ‘migration ellipse’, while recognizing that it is only an ellipse for constant velocity). To migrate any constant-offset section, this operation must be done for every data sample on the section. Alternatively, migrated output values may be obtained directly by summing over curves on the constant-offset section. This summation is defined by the ‘migration summation operator’, whose geometrical shape also depends on traveltime and offset of the source and receiver. In two dimensions, for non-zero-offset sections the summation is over vertical slices through the surface of a Cheops' Pyramid (Claerbout 1985). In variable-velocity media, ray tracing may be used to find the migration summation operators (McMechan and Fuis 1987).

In addition to the geometrical considerations, weighting and spectral modification factors must be applied to provide a solution consistent with the wave equation and Kirchhoff diffraction theory (Schneider 1978). Rigorously applied, the weighting function across the migration operator for non-zero offset can be complicated, but often a simpler function will suffice (Stolt and Benson 1986). In practice, a truncated migration summation operator is usually applied, leading to savings in computer time (Hood 1981).

The resulting migrated constant-offset sections may then be stacked, with residual normal moveout corrections applied if necessary (Gardner, French and Matzuk 1974). Prestack migration can, in principle, easily be extended to three dimensions, and could be applied even with media of variable velocity.
Directional deconvolution and implementation

Referring again to Fig. 3b, the take-off angle at the source determines the position on the migration ellipse. There is also a one-for-one correspondence between take-off angle and location on the migration summation operator for each point on the migrated output section (Fig. 4). If the constant-offset section is now deconvolved for the source signature radiated at one take-off angle, then data samples on the deconvolved section should only be included in each migration summation operator at the appropriate point for that take-off angle. Thus, to perform the directional deconvolution, each constant-offset section must be deconvolved separately with a full range of filters for the source signature radiated at different take-off angles; then the migration summation operator must sum across the set of deconvolved sections, selecting the section which has been deconvolved for the correct source signature at each point. This method of directional deconvolution is thus inextricably bound up with the prestack migration process.

To implement the method, a suite of deconvolution filters must be designed for the far-field source signatures radiated at different take-off angles. A discrete increment in angle for successive filters must be chosen, and also the maximum angle to the vertical which needs to be considered.

Source directivity increases with frequency. So the amplitude spectrum of the desired output for the signature deconvolution must be chosen to avoid excessive amplification of high frequencies at the larger take-off angles.

In prestack migration, the usual practice is to calculate each migrated output sample in turn by using the whole migration summation operator, with the chosen weighting function, over the constant-offset section (Jain and Wren 1980). If this were done with the directional deconvolution scheme, the output of every deconvolved version of the constant-offset section would need to be stored. To avoid this storage problem each of these sections can be considered in turn, and its contributions to the migration summation operator for each point on the migrated section can be calculated. The migration summation operator may be truncated by considering only a limited range of take-off angles.

In our results shown below, the increment in take-off angle for successive signature deconvolution filters was chosen to be 4°, with angles up to 38° from the vertical for the shortest offset and angles up to 54° from the vertical for the largest offset, these angle ranges being chosen to migrate the maximum dip present. Weighting of the migration summation operator was limited to a factor given by the cosine of the take-off angle, which had very little effect, and an additional cosine taper to
reduce the weighting smoothly to zero at the limiting take-off angles.

The 2D spectral modification factor is a weighting of the amplitude spectrum according to the square root of the frequency and a phase lag of \( \pi/4 \) radians at all frequencies. This was applied after summation, with a bandpass filter.

**Physical model data**

The ultrasonic seismic modelling system at the University of Durham is designed to acquire reflection data in common midpoint gathers from solid models immersed in a water tank (Sharp, Peacock and Goulty 1985, presented at 47th EAGE Meeting, Budapest). The dominant frequency in the signals is around 400 kHz. The directivity of the source transducer is similar to that of a marine seismic source array for their respective signal bandwidths (Fig. 5a). It is measured by placing the receiver in the far field and pointing it directly at the source while the latter is orientated in different directions. The sensing element in the receiver is a piezoelectric disc of 1 mm diameter which also has some directivity, but much less than the source transducer and so we have ignored it.

A zero-phase wavelet was chosen as the desired output for directional deconvolution of the source signature, with an amplitude spectrum designed to be a compromise for signatures radiated over the range of take-off angles. Optimum-lag Wiener shaping deconvolution was used to design the suite of filters. The effect of this deconvolution is to synthesize a source which radiates the same zero-phase wavelet in all directions, but with a reduced amplitude at larger take-off angles, similar to the real signatures. This dependence of the amplitude on take-off angle was maintained because of the low signal amplitude, and consequent poor signal-to-noise ratio, at large take-off angles.

The results of directional deconvolution of the source signature are contrasted in Fig. 5b with the results of signature deconvolution using the vertically travelling waveform.

A dataset was acquired over a perspex wedge model suspended in water, with the transducers at a sufficient depth to ensure that there was no interference from the ghost reflection at the water surface (Fig. 6). The top of the perspex sheet was 11.5 cm below the transducers and 2.1 cm thick. The sloping face of the wedge has a dip of 27°.

Two hundred four-fold CMP gathers were acquired, at a CMP spacing of 0.1 cm, with offsets of 1.5, 2.5, 3.5 and 4.5 cm, so the target depth is over twice the maximum offset. Using a scaling factor of 20000, applied to distances, traveltimes and frequencies, this model experiment simulates a seismic reflection profile with dimensions appropriate for oil exploration: target depth between 2 and 3 km; streamer length of about 1 km; and data bandwidth centred on 20 Hz.

The constant-offset section for the offset of 3.5 cm is shown in Fig. 6b. The reflection from the top of the perspex is horizontal at the right, and from the sloping face of the wedge it is displaced down dip to the left.

The four constant-offset sections were individually migrated, using a constant velocity of 1480 m s\(^{-1}\), both with directional deconvolution and with signature deconvolution using the vertically travelling source waveform. The results of migrating the constant-offset section of Fig. 6 after each type of signature deconvolution are shown in Fig. 7. The horizontal reflection at

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**Fig. 5.** (a) Source signatures from the ultrasonic transducer at different take-off angles. (b) Results of deconvolving the source signature using a deconvolution filter designed from the vertically travelling wavelet (left), and from the wavelet recorded at each take-off angle (right). The trace length is 32 \( \mu \)s.
The top of the perspex is adequately deconvolved on both sections, but only has the correct phase after directional deconvolution. The reflection from the sloping face of the wedge comprises larger take-off angles from the source, and the directional deconvolution shows a distinct improvement.

The final four-fold stacks of the migrated constant-offset sections are shown in Fig. 8. The section resulting from directional deconvolution shows higher resolution and better reflector continuity. The first reflector is the top of the perspex and is correctly positioned. The second reflector, with about half the time-dip of the first,
is the primary reflection from the horizontal base of the perspex. It is still affected by velocity pull-up because prestack migration was done with constant velocity. The third event, which dips slightly to the right, is the mode-converted reflection from the base of the perspex which travels as a shear wave within the perspex. Thus its amplitude is only significant below the wedge slope. Note that the polarity of the second and third reflections in Fig. 8b is the opposite of the first from the top of the perspex, as would be expected. The noise beyond the toe of the wedge is background noise which has been amplified by normalization of the energy in each trace before stack.

Discussion and conclusions
We have proposed a method for directional deconvolution of the source signature from marine seismic source arrays in combination with the prestack migration process. It may be extended to arbitrary accuracy by reducing the increment in take-off angle for calculation of the signature deconvolution filters and increasing the accuracy of the migration summation operator, subject to knowledge of the velocity field. We have illustrated the technique on physical model data, using prestack migration with a constant velocity, where it appears to have worked successfully.

To reduce dependence on knowledge of the velocity field, the directional deconvolution could be combined with the DMO process (Deregowski and Rocca 1981) because location on the DMO operator also has a one-for-one correspondence with take-off angle at the source. Circumstances where DMO is desirable (i.e. steep dips) are just the circumstances where attention may need to be paid to directional deconvolution.

The method may readily be applied to 3D data. It could also be extended to correct for receiver directivity, because location on the migration or DMO ellipse has a one-for-one correspondence with the raypath angle at the receiver as well as at the source. However, each constant-offset section would have to be deconvolved a great number of times as raypath angles at source and receiver were incremented independently.

Further work needs to be done to evaluate the effectiveness of the method on real source arrays.

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Fig. 8. Four-fold stack of migrated constant-offset sections. (a) with conventional signature deconvolution, (b) with directional signature deconvolution. Arrows mark the extent of the wedge slope. Traces were normalized to constant energy before stack.

References

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