A novel warped-space concept for interactive 3D-geometry-inversion to improve seismic imaging

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It is a commonly accepted truth in the oil industry that ‘the easy oil has been found’. Finding the remaining hydrocarbons requires better technologies. Examples are exploration projects below salt and basalt, which are difficult to image with seismic. The main exploration method is still seismic but it has become more important to integrate seismic with other methods in order to improve imaging. In areas of strong lateral velocity and density changes, gravity modelling can help to improve velocity models used for seismic imaging. Efforts of joint interpretation of e.g., seismic, gravity and EM methods lead to more and more realistic and therefore more complex models.

At first the word ‘complex’ appears only as a rather fashionable replacement for ‘complicated’. ‘Complexity’ characterizes a system that is difficult to overlook, but where profound analysis allows the decomposition into sub-units, e.g., an analysis of the ‘entanglement’. By dealing with single parts and understanding their system-behaviour, managing complexity can become possible. A joint interpretation on the other hand can be named emergent modelling. This more holistic approach leads to more insight than analysing all single aspects separately. The goal should therefore be to take advantage of emergent effects by modelling different aspects simultaneously.

3D gravity and magnetic field and full tensor modelling are used to improve the results of seismic imaging projects. This applies especially to areas of strong lateral velocity and density contrast with corresponding imaging problems. Typical areas where gravity and magnetic modelling have been successfully used are sub-salt (for example O’Brien et al., 2005; Fichler et al., 2007) and sub-basalt (e.g., Reynisson et al., 2007).

The integration of human geo-expertise, different techniques and different geo-disciplines play another important role. This aspect becomes particularly manifest in interactive modelling. In the forthcoming chapters we describe a novel user-software-interaction, automated interpretation and hybrid techniques where triangulated facets and voxel-cubes are treated in parallel. These ideas allow integrating models for different data such as seismic, gravity, magnetic and EM (planned). This approach in itself is complex but it allows us to address nature’s complexity better than before.

Modelling vs inversion – integration is needed

In general, a distinction is made between forward modelling and inversion. An example of forward modelling is the solution of differential equations under the assumption of constraints and/or initial conditions in a region whose geometry and physical properties are well known.

In theory, the Earth-body can generally be interpreted as a data producer (e.g., gravity) or as a filter, which can receive data (e.g., seismic waves), changes these data and outputs them as a product of the filter-process. The generated and/or output-data are measured geophysical data. The solution of the inverse problem, however, deals with measured and processed data to derive the physical properties and structures in the Earth-filter.

Mathematically, inversion of geophysical data is always an ill-posed problem because it usually suffers from ambiguity. Within the experimental accuracy many different models ‘cause’ the same data. This ambiguity can be – and must be – reduced by constraining a priori information, however ambiguity can never be completely ruled out. Therefore, independent data should be interpreted in a joint approach.

Data inversion is a central step in processing and interpretation of measured data (among many others references e.g., Clauser, 2014). Through the inversion of data, the Earth-filter process is reversed. The aim is the determination of the characteristics of the filter. This holds also for data intrinsically produced by the Earth-body itself like e.g., gravity or magnetism.

Various inversion methods are used to interpret geophysical data – some 4D (e.g., time dependent stress modelling), some 3D, most still 2D, and some even 1D. Examples include seismic 2D-raytracing models, 2D and 3D density modelling.
as well as 1D/2D electromagnetic modelling. These modelling procedures are often restricted by single physical parameter interpretation due to limited hard- and software capabilities.

Today in the oil and gas industry both modelling and automated inversion software are used mostly separately and it is still hard or even impossible to include constraints in modelling and/or inversion processes. The method tightly integrates interactive modelling and inversion for 3D models. The user always stays in control and can take constraints into account. This novel concept of ‘interactive 3D inversion’ has been implemented in the software platform IGMAS+ (Interactive Geophysical Modelling Assistant, Lahmeyer et. al., 2010; Alvers et al., 2013).

Interactive modelling – a great step towards exploring possible scenarios

When it first became possible to use computers to interactive-ly change two-dimensional model geometries of gravity and magnetic models around 25 years ago, a new way of exploring different geological interpretations was reached. Before, the loops of ‘change geometry and recalculate’ (forward modelling) took hours for even simple models containing only few geological structures. Also, the next level of interactivity, where the user gets real-time feedback for changes while e.g., dragging geometry points with a computer mouse, brought another quantum leap. This also led to more realistic, more complex models in much shorter time since many variants can literally be explored in real-time.

But still the limitations of 2D modelling were and still are huge, because modelling in 2D is nearly never meaningfully applicable for realistic geological structures. The assumption that a 2D-XZ-section is infinitely extended into the Y-direction is a wrong pre-condition in (probably) all cases of real nature. Also, so-called 2.5 or 2.75-D approaches are in many cases not good enough to model subsurface structures realistically.

Modelling by using (fixed) 2D sections of a 3D model (Götze and Lahmeyer, 1988) also limits flexibility to shape three-dimensional geo-objects. Creating or changing general 3D structures on the other hand has other problems. It requires the right, intuitive, easy-to-use tools for creation of realistic geological structures. Transferring tools from e.g., the cartoon animation industry was of limited help. Free form deformation (Sederberg and Parry, 1986) was helpful to some degree, but not perfect (Alvers, 1998). With these better modelling tools and the ever-increasing computing power, models became more and more realistic in terms of geometrical resolution. The number of structures and therefore the number of e.g., triangles and/or voxels and the resolution of grids increased significantly. The need for new techniques fostered the development of novel approaches e.g., shown first in Gocad (Mallet, 1992).

In 3D-gravity modelling most commercial packages allow forward calculation of models based on single valued grids with constant densities or voxel-cubes in between layers or based on voxel-cubes allowing more flexible density distributions. Differences between modelled and measured gravity can be analysed in order to derive where and how the model needs to be updated to improve the fit, as well satisfying information from seismic and other methods. When it comes to updating these models things become more difficult in 3D. Simply editing the models interactively like in 2D is usually not possible. Especially if more than one measured field has to be fitted. It is very difficult to gain an overview of the coincidence of various tensor components (full tensor gradients – FTG), invariants, and eventually in addition the magnetic field simultaneously. Interactivity benefits from the help of automated strategies to invert various field components. Appropriate tools are not available or are very cumbersome to use. But interactivity needs the help of automated strategies to invert selected components or combinations of fields and their components.

Improved model geometries – hybrid models

Instead of providing either voxel or grid-based and triangulated facet models it has become more common now to use hybrid models (Figure 1) where both model types are combined (Schmidt et al., 2011). Single value grids are used to define simple density boundaries like sea bottom or top base- ment. Voxel-cubes can then be used to describe more complex density distributions in selected bodies. These can for example be derived from known density-depth relations or from velocity cubes converted to density. Complex geometries like salt domes with overhangs can be described with simple grids as well, but using multi-z grids is much easier. Hybrid models including multi-z surfaces have been implemented in IGMAS+. Hybrid models are quite often also used in seismic imaging such that model exchange between imaging applications and IGMAS+ has become quite easy. Please note that for the pure forward calculation single or multi-valued grids are not really needed. The whole model could be calculated from a voxel-cube. However, in case the model geometry shall be modified describing the main interfaces by single and multi-valued grids is more convenient.

In case hybrid models shall be modified in interactive modelling workflows we need:

- Very fast forward modelling algorithms and 3D display.
- A flexible and easy-to-use 3D-editor.
- Integration with data and information like seismic sec- tions, results from MT-inversions, well markers, logs and geological maps to guide the interactive modelling process.
- An inversion algorithm for geometry changes supporting and enhancing interactive workflows.

Needs for modelling tools – beyond gravity and magnetics

Today’s geophysical interpretation requires an interdisciplinary approach, particularly when considering the available amount of state-of-the-art information contained in comprehensive
databases. A combination of different geophysical surveys employing seismic, gravity and electromagnetic together with geological and petrological studies, can provide new insights into the formation and tectonic evolution of the lithosphere and natural deposits. Interdisciplinary interpretation is essential for any numerical modelling of these structures and the processes acting on them.

In the past few years the turnaround time of depth-imaging projects has also became much shorter. Instead of days or weeks, a pre-stack depth migration is now often carried out in less than an hour. This demands that the turnaround time of gravity modelling projects also needs to be improved in order to be integrated in the depth-imaging process (Lahmeyer et al., 2010).

This is particularly true for 3D modelling of sedimentary basins, which are often underlain and permeated by huge salt structures. Although the new modelling tools and algorithms which are described in this paper can be applied to nearly all cases of potential field modelling (e.g., in mining, pure geophysics on very different scales) here we will focus on the 3D modelling of salt structures in the following due to their importance in the interpretation of sedimentary basins in oil and gas industry.

The suggested workflow for salt-related modelling is as follows:

- Describe the salt body by a multi-z grid. Such a model can quite often be delivered directly from imaging applications. In case this is not possible a triangular grid can be (automatically) wrapped around the salt body described by voxels.
- Convert the sediment velocities to densities. This conversion is possible and conversion factors are often derived from well data.
- Fill the area of the voxel-cube where salt is assumed with sensible sediment density values. These can either be derived by 3D interpolation or also by simply using a density depth function derived from a nearby well. These densities are needed when during the modelling salt needs to be replaced by sediment densities.
- Embed the salt body described by the multi-z grid with constant salt density inside within the sediment model described by voxels.
- Forward calculate the hybrid model and optimize the salt geometry to fit the gravity data. Depending on the depth of investigation we either use normal gravity, FTG data or a combination.
- Feed the updated salt geometry back to the depth-imaging process.
- Alternative models can be generated based on different constraints, and seismic imaging should test all these scenarios.
- The best model is further improved by seismic imaging and fed back to gravity modelling.
Against this background, we were working towards a 3D interactive software tool, which eases the interpretation of gravity and magnetic data. The software integrates geophysical models, information and data from both geophysics and geology. Dealing with hybrid models and many different fields requires high-performance algorithms in order to shoot for real-time responses.

**Performance improvements – speed-up for interactivity**

For interactive work it is essential that, like in 2D, after model changes the recalculation of the model is done immediately, ideally in less than one second. This can be achieved by taking the following six aspects into account:

- **The first aspect is to implement more efficient algorithms.** Ideally, like in 2D, all modelled fields are updated in real-time while the model is altered in order to allow a broad exploration of hypothesis. For realistic models this is still not fully possible. Response times of one second are achieved on a rather powerful desktop computer. It is important that this performance is achieved not only when a single point in the model is altered but also when bigger parts are changed at once (e.g., several triangles).

  - The basis for a fast recalculation is a changed-only-recalculation (fortunately possible in gravity and magnetics):
    - Identify these parts of the model which have been changed,
    - Subtract these effects from the actual field,
    - Add the newly calculated effects.

  - The second aspect of fast calculations is smart data handling for voxel-cubes in order to identify those voxels, which have changed (parameters like density, susceptibility and/or resistivity) after geometry changes. In case of e.g., a salt dome model, the sediment densities are described by voxels, the salt by a (often) constant density. The salt-sediment-boundary is moved to generate more salt volume and the changed voxels of sediment have to be identified, subtracted as sediments and replaced with the density of the salt dome. A tree structure (Finkel and Bentley, 1974) is used to speed up the search through millions of voxels.

  - The third aspect is the speed of the calculation of the gravity effect for triangles (single- and multi-z surfaces) and voxels. This can be achieved for example by parallelizing calculations on all available CPU-cores and/or GPU via OpenCL (Khronos Group, 2011). Another way of speeding up the calculations is to approximate them. For deeper parts of the model, the calculation of the voxels can be replaced by point masses. Gaussian Quadrature can approximate the exact calculation of the surface integrals over the triangles, if certain conditions hold. Approximations have to be applied carefully in order to prevent errors. For the recalculation of the model (after model changes) this is often less critical. The introduced error of subtracted and newly added calculation are quite often very similar and cancel out each other at least partly. Parallelization and approximation can obviously be combined. The usage of FFT for the calculation of the voxel model can speed up the calculation of the whole model as well. However, for local recalculation the FFT is less optimal since the whole model must be recalculated. Depending on the available hardware educated choices on what algorithms should be applied has to be made.

  - The fourth aspect of performance improvement is to get fast updates of the complex 3D graphics. Current top-end graphic cards and good graphics libraries solved this problem through hard and software. Also a new concept was developed for optimizing 3D model geometries consisting of voxels, grids and triangular facets.

  - Aspect five is model resampling. By keeping the shape of a highly resolved e.g., triangle interface this algorithm (Garland and Heckbert, 1997) allows – depending on an user-given threshold – for the reduction of the number of triangles by factors of 10 to 100.

  - A sixth aspect is cloud computing. Providing maximal flexibility to use available resources in the local intranet and the connection to cloud providers enables users to perform calculations in much shorter timeframes with – in the case of a commercial cloud provider – often limited financial resources.

**A new 3D editor – keep model-topology**

As explained above in previous software versions of IGMAS+ (Götze and Lahmeyer, 1988; Schmidt et al., 2010) the 3D editing focused on vertical, parallel sections. This was easy-to-use but caused several restrictions like incompatibility with Gocad grids. IGMAS+ interfaces could be exported to Gocad’s T-Surf format, but importing Gocad models for example from seismic imaging applications required resampling the interface such that all the corner points of the triangles where within one of the sections and that triangles did not cross these sections. Another problem was that moving single points especially in the deeper part of the models did not generate visible gravity changes. Moving a collection of neighbouring points overcame this problem partly, but in cases where the sections were spaced closely, the gravity effect was still quite often marginal. Only after updating several points in several sections was the gravity effect visible.

These limitations have been overcome by the introduction of a new type of 3D editor, which is based on the concept of warping the space (Alvers, 1998). A 3D grid (lattice) of suitable size, resolution and orientation is placed (by the user) in the area where the model shall be changed. By moving the points in the lattice a distorted room is generated and the same distortion is applied by tri-linear interpolation to the model geometry in the lattice. See Figures 2a to 2c.

Coarser grid spacing of the lattice results in more regional updates for example for deeper parts of the model.
Finer lattices would allow for more detailed modelling in the upper parts of the model. Moving the nodes of the lattice can either be done in 3D or in 2D sections, aligned with a section of the lattice. The latter might be very useful when seismic sections are used to guide the modelling. A section of the lattice is aligned with the seismic section and then the points of the lattice within the seismic section can be moved within the plane of the section. Tests have shown that full 3D editing is best done in stereo, preferably with a 3D pointer.

Applied to a hybrid model the distortion is only directly applied to the grids (conventional and multi-valued grids). The voxel-cube is not distorted. Only when the density values in the voxel-cube use an updated surface as reference (for example density depth function referenced to the sea bottom) the voxel-cube is updated. Applying the distortion to the voxel-cube might in some cases be useful as well but has not been implemented yet.

A big advantage of an editor based on this concept is that warping (or distorting) the space – rather than ‘touching the model itself’ does not change the topology of the model ‘behind’. In other words, if model elements (e.g., triangles) do not cross each other in the undistorted model (space), they won’t cross each other in the distorted model either. This makes it the ideal tool for inversion algorithms for optimizing the model-geometry. Therefore, it was a consequent step to expand the software by a 3D inversion.

3D interactive inversion – set the expert in the driving seat

Even with a sophisticated editor updating a 3D model can be cumbersome. The above-mentioned topology-conserving approach of space-warping allowed implementing a geometry inversion, supporting an interactive workflow.

As ‘optimization-daemon’ we decided on Covariance-Matrix-Adoption Evolution-Strategy CMA-ES (Rechenberg, 1973, Hansen and Ostermeier, 2001), the strongest non-linear inversion technique to our knowledge and our own experience (Alvers, 1998; Hansen et al., 2010). The only drawback is that a few algorithmic constraints must be respected in order to make the self-adopting work. Where in interactive modelling software-bounds-checks of node-movements can avoid topology changes this cannot be done when CMA-ES is applied, without destroying self-adopting, the core idea of the method. Such corrections (coming from outside the CMAES) interfere with the learning. Therefore the lengths of the grid-legs between the nodes are targeted as optimization parameters and not the node-coordinates themselves. The advantage here is that the only constraint is greater-than-zero-length of a leg. The disadvantage is that grid-legs are still not independent of each other, which, again, negatively influences the self-adopting-learning-mechanism of CMA-ES. The solution is the introduction of a rather complex finite-differences approach, which cannot be described here in depth.

Inversion can either be run over the whole model, but typically it is used in smaller parts of the model, helping to solve local problems and/or proving/disproving local hypotheses. It is also possible to exclude parts of the model (for example top salt) from the inversion.

The efficiency of the CMA-ES algorithm is very good in terms of stable convergence because the 3D editor described above guarantees the topological validity of the model, such that continuous checks for crossing triangles are not necessary.

The workflow for the interactive inversion can be described like this:
- Lattice creation: The user defines a box around the parts of the model interest as described above.
- An automated ‘daemon’ can be started with the task to alter grid nodes (inside the box only by default) in order to optimize the model for measured and calculated fields (gravity components, FTG, magnetic components and remanence if chosen).
The process is monitored by the user e.g., by looking at a slowly rotating model.

- The user can stop the ‘daemon’ if the changes start becoming unrealistic.
- Rewind the process if overshot already.
- Readjust constraints, reposition the lattice, change resolution by increasing or deleting cells into the lattice etc. and start/continue the ‘daemon’ anew.

One prerequisite of the described combined approach of space-warp and CMA-ES is a very powerful strategy to speed up forward calculations. First, due to ever-finer and more complex models and second, due to the fact that the CMA-ES explores the parameter space with populations of (model- individuals with the advantage of increased probability of global convergence, but also at the cost of necessary additional model evaluations if compared to gradient methods.

**The new concept applied to the SEAM model – promising results**

The concept of the interactive inversion has been tested using the synthetic SEAM Model (Pangman, 2007), which is based on a complex salt structure, similar to those found in the Gulf of Mexico. We changed the model in the area of an overhang of the salt dome, simulating the effect of incorrect seismic imaging. This incorrect salt model was placed in a simplified sediment model, described by a voxel-cube.

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**Figure 3a** True SEAM model in 3D (without sediments). The “Well” was used to calibrate the velocity – density transformation.

**Figure 3b** True SEAM model in a 2D section with correct sediment model and measured gravity.

**Figure 3c** Modified SEAM model in a 2D section with simplified sediments and measured and modelled gravity. This model and the corresponding residual anomaly is the starting point for the interactive inversion.

**Figure 3d** Salt geometry as recovered by the interactive inversion in a 2D section with measured and modelled gravity. The original salt geometry has not been recovered perfectly, but using this improved model should improve the seismic imaging.
This sediment model was derived from a smooth velocity model, converted to density. Consisting of about 260,000 triangles and 25 million voxels, this model can be regarded as very realistic. The residual anomaly is explained by the incorrect salt model (modified overhang) and the simplified sediment model. Also, this situation can be regarded as very realistic. The interactive inversion was applied to the area of the salt overhang, assuming that the top and base of the salt are known (Figure 3a). The inversion could recover the original salt geometry to a large degree (compare Figures 3b and 3d). The recovered model is of course not perfect, but would be a good base for the next imaging step. Under the assumption that the base of salt is uncertain, the solution found is of course different, reflecting the ambiguity of potential field data. In a real case, both models would have been given back to imaging to test which model works best for imaging (scenario testing).

In this example the first vertical gradient has been used for the inversion. Other components of the gravity field and magnetic measurements can be included in the inversion as well. Doing so depends on extremely fast forward calculation algorithms as described.

Conclusion
Interactive 3D modelling has now become feasible through

- Flexible model structure combining triangle facets, single
  and multi-valued grids with voxel-cubes (hybrid models),
- A novel 3D editor using the concept of warping the space
  and
- Enormous performance improvements.

The interactive work is supported by an automated inversion, integrated in the interactive workflow. Therefore we call it interactive inversion. The application of these new tools on the very realistic SEAM model demonstrates the usefulness of these tools. An advantage of the presented concept is that the tools are topology preserving.

Approaches for joint data inversion of different geophysical methods have been discussed for decades. Even today there are no software packages available, dealing with e.g., seismic, EM, magnetics and gravity simultaneously in an elegant way. Therefore our aim will be to develop tools able to do joint inversion of different datasets as described in this article.

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