Non-linear behaviour for naturally fractured carbonates and frac-stimulated gas-shales

Nick Barton

Abstract
Gas-shales and naturally fractured reservoirs usually produce from several kilometers depth, with fracing-stimulation and eventual water-drive respectively. Due to porosity, the matrix is generally weaker than is typical for basement rocks. The potential pore pressure reduction of tens of MPa during the early life of the fields, may therefore be a significant proportion of the strength of the matrix. Inevitable non-linear rock strength behaviour for the matrix should not then be ignored. It is therefore unrealistic to utilize a linear Mohr-Coulomb strength criterion as so frequently seen. The joints or natural fractures in the shales and carbonates, which are so important for production, will have producing fracture sets with different roughness and aperture, and few of them are planar enough to follow the frequently used linear Mohr-Coulomb behaviour. Non-linearity especially applies to the favourable shear strength-dilation-permeability coupling which is relevant for both NFR and gas-shales, and to the less desirable stress-closure-permeability coupling of a stress-sensitive reservoir. Non-linear constitutive modelling, partly based on the joint- or fracture-roughness coefficient (JRC) used widely in rock mechanics, also applies to the conversion from hydraulically interpreted theoretical smooth-wall apertures (ε) to the larger and non-planar-non-smooth-wall physical apertures (E) through which the oil or gas actually flows to the wells. Simple index tests which can also be applied on joints or fractures recovered in occasional and inevitably expensive core, and which can also be estimated when mapping fractured pavement analogues, have been available in rock mechanics for several decades. They were already incorporated in coupled distinct element (jointed) non-linear modelling routines in 1985. However, their implementation in petroleum industry geomechanics seems to be very rare judging by numerous workshops attended in the last seven to eight years on both sides of the Atlantic. Application of non-linear (non Mohr-Coulomb) rock mechanics, using recovered core from Ekofisk in 1986-1987 in order to model fracture shear-dilation coupling with simplified E and ε tracking during compaction, may be the earliest example.

Introduction
In the EAGE workshop on Naturally Fractured Reservoirs in Real Life, held in Muscat, Oman in December 2013, Price and Wei (2013) from Shell reported on the conclusions from retrospective analyses of eight case studies of fractured reservoirs, mostly related to carbonate reservoirs in Oman. Through extensive fracture network modelling by a large team of company collaborators, the authors identified the most important factors which they considered necessary for improved history matching. A production and forecast time-scale of 10 to 40 years was considered in this extensive study by Shell.

Their list of important factors, including their own verbal additions given during their lecture, included the following: fracture volume, fracture density, fracture clusters, permeability anisotropy due to stress, and aperture sensitivity to stress. Most of these factors would seem to be amenable to realistic conceptual modelling, using rock mechanics principles and available non-linear methods. Such modelling would obviously need to be made at reduced scale at first, using coupled-process distinct element methods, such as the two-dimensional distinct element (jointed) model UDEC-BB (with non-linear Barton-Bandis joint behaviour), or the three-dimensional model 3DEC-MC (with less realistic linear Mohr-Coulomb joint behavior). The detailed behavioural trends thus revealed would subsequently need to be up-scaled but not lost, ready for potentially improved reservoir simulator modelling.

Geomechanics and fracture characterization is done differently in rock mechanics
Based on presentations made in the Muscat fractured reservoir workshop, and based on presentations made in several similar workshops and courses attended on both sides of the Atlantic in the last seven to eight years, the writer has gained the strong impression that geomechanics, complex enough as it is, seems to be mostly practised in the petroleum industry without considering non-linear shear strength, dilation and stiffness of the different fracture sets. Much of reality is lost if this is true.

The desirable non-linear shear-dilation-permeability coupling and the less desirable and very non-linear fracture aperture-closure of a stress-sensitive reservoir, due to effective stress change during production, are also apparently not yet a part of open-source petroleum geomechanics literature. The aperture-closure would oppose the assumed benefits of major fracture sets ‘always’ being parallel to the major
The terms, used in the world of tunneling for the last 40 years, describe rock mass quality, joint shearing, and the effects of shearing: see Barton (2007a) and Barton (2013a). In fact it has been known for many decades that shearing is important for production from carbonate reservoirs, for instance at Ekofisk as described from Norwegian studies in Barton et al. (1986, 1988), and more generally the importance of shear stress was described in the USA by Christine Barton et al. (1995), Townend and Zoback (2001) and by Fairhurst (2013). In the last decade, fracture shearing as a result of fracking in gas-shales has to be considered the probable most important and ‘wider-radius’ result of fracturing. The impermeable shales cannot produce without the wider-reaching fracture deformation, especially shearing, which according to microseismic evidence, stretches well beyond the central ‘ellipsoidal’ propped artificial tension fracture region. See for instance Fisher and Warpinski (2011) and Dusseault (2013).

Also based on workshops attended it is evident that the commonly used term fracture characterization as used in the petroleum industry is rather different from the term joint characterization used in rock mechanics for civil or nuclear waste isolation engineering. In rock mechanics, the relative roughness of the fracture sets would be described using JRC (= joint roughness coefficient) and using the wall strength JCS (= joint wall compressive strength) of the different fracture sets. These simply obtained parameters would be an automatic additional focus of attention, and would provide important quantitative abilities to include non-linear shear

<table>
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<tr>
<th>PARAMETER</th>
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<td>The cohesive strength of intact rock (assuming linear Mohr-Coulomb strength envelopes). A non-linear alternative is proposed.</td>
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<td>φ</td>
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<td>r&lt;sub&gt;n&lt;/sub&gt;, R</td>
<td>Schmidt-hammer rebound (mean of top 50% of tests) on respectively the saturated joint or fracture wall, and on the dry unweathered rock. The ratio r/R describes the degree of fracture-surface weathering/alteration.</td>
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<td>L&lt;sub&gt;n&lt;/sub&gt; and L&lt;sub&gt;n&lt;/sub&gt;</td>
<td>Nominal laboratory-sample length, and in situ block size or spacing of cross-joints (cross-fractures). In case of anisotropic fracture spacings, tabular block shapes may require two estimates of L&lt;sub&gt;n&lt;/sub&gt;.</td>
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<td>φ&lt;sub&gt;r&lt;/sub&gt;</td>
<td>Residual friction angle of a joint or fracture after a significant amount of shearing. Ultimate strength is reached earlier, at the end of a shear test. (Typical range of minimum φ&lt;sub&gt;r&lt;/sub&gt; is 24° to 34°).</td>
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<td>JRC&lt;sub&gt;mo&lt;/sub&gt;</td>
<td>The shear-displacement-dependent mobilization (and post-peak degradation) of the joint roughness coefficient. A widely applicable dimensionless model is illustrated in Figure 15. This demonstrates that frictional strength is not so elementary as single (peak values) of μ = 0.6, 0.85 etc. as pioneered long ago by Byerlee and also widely used today in the USA.</td>
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<td>δ and δ&lt;sub&gt;peak&lt;/sub&gt;</td>
<td>The ongoing shear displacement, and the shear displacement needed to reach peak shear strength, which reduces to &lt; 1% of in situ block length when block-size exceeds approximately 100 mm.</td>
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<td>σ&lt;sub&gt;ε&lt;/sub&gt;, σ&lt;sub&gt;1&lt;/sub&gt;, σ&lt;sub&gt;3&lt;/sub&gt;, and σ&lt;sub&gt;n&lt;/sub&gt;</td>
<td>Effective stress, axial (major) stress, confining (minor) stress, and normal effective stress across a joint or fracture.</td>
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<td>K&lt;sub&gt;n&lt;/sub&gt; and K&lt;sub&gt;s&lt;/sub&gt;</td>
<td>Normal and shear stiffness of joints or fractures. Both are stress-dependent and non-linear. K&lt;sub&gt;n&lt;/sub&gt; is also block-size dependent (it is fortunately lower in situ). The ratio Kn/Ks may be from 10 to 100.</td>
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<td>Q-value, Jr, Ja</td>
<td>These terms, used in the world of tunneling for the last 40 years, describe rock mass quality, joint roughness and joint alteration or clay-filling. Q correlates to the stress-dependent static deformation modulus E&lt;sub&gt;max&lt;/sub&gt; (resembling Q&lt;sub&gt;P&lt;/sub&gt; in GPa), and to the stress-dependent P-wave velocity V&lt;sub&gt;p&lt;/sub&gt; (see Barton, 2006, 2007c). Q&lt;sub&gt;UWS&lt;/sub&gt; approximates the depth dependent permeability of a rock mass. Usual range in top 1 km is 10&lt;sup&gt;-4&lt;/sup&gt; to 10&lt;sup&gt;-10&lt;/sup&gt; m/s.</td>
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EAGE’s 2013-2014 president Gladys Gonzalez recently expressed her intention of focusing EAGE activities on ‘Making a difference on a global scale by narrowing the gap’. Her expressed purpose was narrowning the gap between geoscience and engineering disciplines. On a modest scale, this article tries to support these important and potentially oil-and-gas producing objectives.

**The shear strength of intact rock may be highly non-linear**

Petroleum production tends to occur from reservoirs typically several kilometres deep, and from rocks with significant porosity and therefore reduced strength in relation to basement rocks of low porosity. Here, one can think of the principally occurring and producing sandstones, carbonates (including chalk) and stimulated oil-shale/gas-shale reservoirs, plus the well fractured-and-faulted basement reservoirs in altered granites, whose productive fractures have very high porosity, despite the usually misleadingly low average porosity, which is often < 1%. Figure 1 shows the non-linearity of dry carbonate rocks, over a large range of stress. Non-linearity will also occur over the lower range of reservoir stresses, since many tens of MPa are involved during the hoped-for life of the producing reservoir.

It can be highly unrealistic to assume linear Mohr-Coulomb \((c + \sigma \tan \varphi)\) strength envelopes, when large potential increases in effective stress occur, perhaps ranging from 20 to 40 MPa, as a result of production. (Note: \(c\) = cohesional strength, \(\varphi\) = frictional strength, and \(\sigma\) is the effective stress; see second page of this article). Furthermore, the relationship between conjugate-fracture intersection angles must be expected to change radically, when progressing from

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**Figure 1** A wide-ranging review of the shear strength of intact rock reported by Barton (1976), included this data set from Mogi (1966). These high-pressure triaxial tests on dry carbonate rocks, and numerous other data from Byerlee (1968) gave the writer the idea of a ‘critical state’ approach to shear strength limits. The critical state line added by the writer, has a gradient of \(\tan^{-1} \frac{1}{2}\), or 26°. The explanation of this term will become clearer in Figure 2.

**Figure 2** The critical state concept suggested by Barton (1976). This has recently been used by Singh et al. (2011) to derive the equations giving the correct deviation from linear Mohr-Coulomb. A few triaxial tests at low confinement give the complete strength envelope. An especially interesting result of their study was that a majority of rock types have \(\sigma_3 \approx \sigma_c\). In other words Mohr circles #2 and #4 are touching or nearly touching, as indeed illustrated in Figures 2 and 3. Note the intermediate stress-circle #3 representing the brittle-ductile transition. This may be reached as effective stresses increase in the weaker reservoir rocks like chalk and shale. Water flooding in chalk would be needed well before this damaging stage.
shear strength behaviour of intact rock over
a near-surface pavement or cliff, to fractured reservoir depths of several kilometres.

Both of these aspects are illustrated in Figure 2, and also in Figure 3. As pointed out by Barton and Pandey (2011) and a few others in the past, it is also unrealistic and often incorrect to add $c'$ and $\phi$ tan $\phi'$, as if these Coulomb shear strength components are mobilized at the same strain. See Barton (2011) for further discussion of this important topic, which was raised more than 50 years ago, in the early days of rock mechanics by Müller (1966).

The maximum possible shear strength, operating when the strength envelope becomes horizontal at the critical state, is expressed as: $\sigma_{\text{max}} = 3 \sigma_{\text{critical}}$. The actual closeness of the uniaxial compression (#2) and critical state Mohr circles (#4) was discovered by Singh et al. (2011). The marked superiority of their 2011 ‘Singh-Singh’ criterion, which also caters for poly-axial stress states, was recently confirmed by subsequent review work presented by an independent group: see Shen et al. (2013), who were presenting a ‘competing’ but less well-fitting intact rock strength criterion of their own, using the results from more than 1300 triaxial tests.

Note that the brittle-ductile transition shown in Figure 2 is reached at an intermediate confining stress, and there is reason to believe that this will be reached in weaker reservoir rocks such as chalk, and perhaps in weaker (more clay-bearing) shales as effective stresses increase. However, pore collapse is a complicating factor in the case of chalk, and a further source of non-linearity.

A conveniently simple ‘linear’ equation, which describes the non-linear shear strength behaviour of intact rock over a range of confining pressure from zero (uniaxial), through the brittle-ductile transition, and up to the critical (maximum strength) state was derived by Barton (1976), but appears to have been over-looked up to now. It had the following simplest possible form, and is repeated in Barton (2014):

$$\frac{\sigma_1 - \sigma_3}{\sigma_3} = M \frac{\sigma}{\sigma_1} + 1.0$$

The respective gradients $M$ for Solenhofen limestone (as in Figure 3), Oak Hall limestone, Nahant gabbro and Westerly granite, as tested and reported by Byerlee (1968), showed a logical progression of 3, 7, 9 and 30 as strength increased. Reservoir rocks will clearly be in the lower range of gradients $M$.

Shearing along fractures or joints is a non-linear process

The intended focus of this paper is the potential contribution of rock mechanics understanding to the changing-over-time production from naturally fractured reservoirs. For instance, see the emphasis on fracture shearing in Barton (2006, 2007a, 2007b). Shearing during continued production may be an inevitable condition for continued production. In addition we will see possible rock mechanics contributions to the ongoing attempt to understand the mechanisms of fracture shear and dilation in stimulated gas-shales. These mechanisms seem to be occurring beyond the central and ellipsoidally-shaped frac-stimulated and propped ‘pod’ region in tight gas-shales and oil-shales. This is because microseismic activity is detected well outside this ‘central’ frac-stimulated and propped region. An excellent introduction to this topic was recently presented by Dusseault (2013). The importance of fracture shearing was specifically referred to by Dusseault, although using linear Mohr Coulomb theory.

An extensive review of fracking stimulation methods and results was given by King (2010), but this was without any emphasis on shearing. The very extensive microseismic monitoring results, such as those described by Fisher and Warpinski (2011) nevertheless give clear evidence of the large extent of shearing mechanisms. They also emphasized the desirable complexity and fracture-including nature of the fracked region, which needs propping where it is without shearing. Shear and dilation, however slight, needs no propping, if the gas-shale is sufficiently brittle, with high enough static deformation modulus and high enough JCS (joint wall compression strength, from Schmidt hammer ‘r’ rebound).

Figure 4 emphasizes the non-linear shear strength of all the strength components of a fractured or jointed rock mass. These strength envelopes are clearly different, due to the non-linearity, to that assumed in most petroleum geomechanics presentations. Only the intact rock, which would represent the ‘intact bridges’ between the natural fractures, has true cohesive and of course tensile strength. The various empirically-derived (a posteriori, not a priori) non-linear...
Figure 4 Comparative shear strength trends for at least four of the components of the shear strength of a rock mass (or fractured reservoir). Only the intact rock (and therefore parts of a rock mass) have tensile and cohesive strength. The two first empirical stress criteria fitted high-pressure test data from Byerlee (1968) and Mogi (1966). The third set of empirical x, y and z parameters are for rock joints with variable joint- or fracture-roughness (JRC) and apply to high pressure (reservoir-depth) or lower pressure civil engineering depths respectively. The joint wall compressive strength (JCS) which can be recorded by Schmidt hammer, is replaced by the triaxially confined strength \( \sigma_3 \) when considering reservoir depths. Filled discontinuities, of relevance to fault zone conditions have much lower shear strengths, and these can be estimated from the Q-system of Barton et al. (1974) which was summarized in Barton (2007c). See latest presentation of the frictional strength J/Jr of the discontinuities such as found in fault zones, in Barton (2013c). (Note that Jr the joint roughness is a rating varying from 0.5 (slickensided/polished) through 1.5 (rough-planar) to 4 (discontinuous). The joint alteration or filling rating Jr varies from 0.75 (healed) through 2 (weathered) to 20 (thickly filled-discontinuity with swelling clay like montmorillonite).

Fracture characterization as practised in rock mechanics

Rock mechanics appears to be very different from the geomechanics and geoscience practised by oil companies, their consultant, and by service companies. There is naturally much more focus on the deformability and strength of rock joints or rock fractures in rock mechanics, as this often determines the stability or otherwise, of potentially unstable opencast mine slopes, road cuttings, dam abutments, rock-wedges in rock caverns, etc. The ability to model these discontinuous processes, also with coupling to conductivity and pore pressure, has been provided by distinct element (jointed) models such as Cundall’s 2D UDEC (universal distinct element code) and by his 3D 3DEC, each marketed by Itasca. These codes have been available for several decades.

An essential step in this deformation-modelling process is the collection of input data for the constitutive joint-
of blocks indicate that the block dimension is more relevant than the joint or fracture length—which of course could be tens of metres. The block-size scaling of JRC and JCS was first given by Bandis et al. (1981), Barton et al. (1985) and in a series of publications stretching through to Barton (2014). The criterion is widely used in rock mechanics, especially for rock slope stability estimation and design, and for input into numerical models for tunnel, cavern, and dam abutment and dam-loading stability calculations.

Joint or fracture shear strength and coupled 4D mechanisms

The tilt tests illustrated in Figures 6 and 7 are used to back-calculate the roughness coefficients JRC illustrated in Figure 8, using the Barton (1973, unweathered) or Barton and Choubey (1977 weathered) non-linear peak shear or fracture behaviour modelling. The most widely used non-linear method, is the Barton-Bandis criterion described in Barton et al. (1985), which has been a part of UDEC-BB since 1985, and has been incorporated in ‘competing’ computer codes more recently. The necessary, but rapidly and cheaply obtained input data, is illustrated in Figures 6, 7 and 8.

Here the widely used terms JRC and JCS are graphically explained by means of diagrams and photographs of the simple index tests, and by roughness profiles (giving the scale-dependent JRC) at different block-size length scales of 100mm and >1m length. (See table of definitions on the second page of this article). JRC applies to block size and not to joint length: in other words it applies to the spacing of cross-joints (in petroleum terminology the spacing of fractures crossing the fracture being characterized). The above is because physical and numerical models involving thousands of blocks indicate that the block dimension is more relevant than the joint or fracture length—which of course could be tens of metres. The block-size scaling of JRC and JCS was first given by Bandis et al. (1981), Barton et al. (1985) and in a series of publications stretching through to Barton (2014). The criterion is widely used in rock mechanics, especially for rock slope stability estimation and design, and for input into numerical models for tunnel, cavern, and dam abutment and dam-loading stability calculations.
Figure 6 A collection of simple index tests for obtaining input data for fractures or joints, using tests on core or on samples sawn or drilled from outcrops. In the left-hand column, direct shear tests are also illustrated, which can be used to verify the results from empirically-applied index tests. Note that the Schmidt hammer should be used on clamped pieces of core to estimate the uniaxial compression strength UCS or \( \sigma_c \). This is done on dry pieces of core (rebound \( R \), use top 50% of results). For the joint wall strength JCS, this is done on saturated samples (rebound \( r \), use top 50% of results). These samples also need to be clamped (e.g. to a heavy metal base). Three methods of joint-wall or fracture-wall roughness (JRC) can be used: tilt tests to measure tilt angle \( \alpha \) (Figure 7), or \( a/L \) (amplitude/length) measurement, or roughness profile matching: the latter obviously more subjective. A standard set of small-scale (length \( L_0 = 100 \text{ mm} \)) roughness profiles, and profiles of >1m (length \( L_n \)), with consequently reduced JRC\(_n\) values, are shown in Figure 8, from Bakhtar and Barton (1984) and Barton (1999).

Figure 7 Left: Tilt tests using slow manual rotation of a 1:200 reduction gear. The joints or fractures can be obtained from drill-core or from sawn blocks extracted from a fracture pavement or from a relevant cliff with jointed/fractured analogue-reservoir-rock exposed. The gravity normal and shear loading acting in the tilt tests brings the joint or fracture to shear failure (angle \( \alpha \), typically 40° to 80° depending on roughness JRC), and this very low stress test (\( \sigma_n = 0.001-0.002 \text{ MPa} \)) is used to back-calculating JRC. One should use saturated joint/fracture surfaces, but not sufficiently wet to cause suction during the slow rotation/tilt testing. Using the Barton (1973) and Barton and Choubey (1977) non-linear peak strength criterion (see equations 2 and 3), shear strength (and shear stiffness) can be estimated at three to four orders of magnitude higher stress, relevant to reservoir depths. In the left-hand photo a Schmidt hammer and a roughness-profiling brush-gauge can also be seen.

Right: Tilt tests on (dry) core sticks for measuring the basic friction angle \( \phi_b \) of unweathered surfaces of the reservoir rock (typically shear failure / sliding at 28° to 32°). The core sticks should be smooth-flat (not polished, nor with ridges). Final preparation can be with sand-blasting. These tests have been used in rock mechanics since their development in Barton and Choubey, 1977. They were used in 1985 and 1986, perhaps for the first time in a reservoir context, in Ekofisk reservoir compaction studies, for characterizing the conjugate joints in the porous chalk. Barton et al. (1986, 1988), Barton (2013a).
Note that the ‘complicated non-linear term’ involving log_{10} provides both the peak dilation angle and the asperity failure component S_A which can be of almost equal (angular) magnitude. When generating shear-displacement-dilation-permeability coupling behaviour, the concept ‘JRC mobilized’ from Barton (1982, 1986) is used. Some simple hand-calculated examples are given later in this paper. These methods may be the key to a quantitative understanding of gas-shale stimulation results. Of course they are also fundamental for production from ‘critically stressed’ NFR. We can move far beyond ‘μ = 0.6, 0.85, 1.0’ by using simple non-linear rock mechanics.

The above fracture roughness and strength parameters JRC, JCS and \( \phi_r \) allow one to generate the desirable shear-dilation-permeability enhancement curves, as probably operating in stimulated gas-shales, and the much less desirable stress-closure-permeability reduction behaviour that makes itself known in stress-sensitive reservoirs, or later on in the life of a producing reservoir. Each of the above occur in the context of potential reservoir 4D behaviour – which is far more interesting and sophisticated than ‘stress and strain’, which is an over-simplification suggested by authors working in a major service company. This type of non-linear, rock mechanics based 4D coupled behaviour modelling...
component, which was termed N, and the shear component which was termed S.

The three basic rock mass load-deformation processes illustrated in the middle of Figure 9 are respectively concave, ‘linear’ and convex. The linearity is due to a ‘cancelling-out’ of the two opposing non-linear tendencies. Each of these three modes of behaviour have been confirmed by large-scale *in situ* testing, including flat-jack loading of an 18m$^3$ (2 $\times$ 2 $\times$ 4.5m) block sawn into columnar basalt at the Hanford-Washington site, one of many studies related to possible high-level nuclear waste disposal in the 1980s.

The UDEC-BB models shown at the bottom of Figure 9 were performed several years later, and help to reinforce the idea of a potentially positive contribution of conjugate jointing.

**Figure 9** The non-linear normal-closure (component N) and non-linear shear-deformation (component S) combine to form distinctive behaviour in the fractured rock mass, with conjugate shearing giving the most impressive potential contribution to NFR and fractured gas-shale productivity. From Bandis et al. (1981, 1983), Barton (1986) and Chryssanthakis et al. (1991). The shear displacement along the modelled rock joints were modelled by UDEC-BB. Use of this code requires input for $JRC_0$, $JCS_0$, $\phi_r$ and relative block-size $L_n$ for the different joint or fracture sets in relation to the test dimension $L_0$. The internal workings of the code automatically provide block-size and stress-dependent shear strength, shear stiffness, normal stiffness, and the coupling of physical aperture (E) and hydraulic aperture (e) where the intrinsic joint or fracture conductivity $k = e^2/12$. Note that P and $\Delta$ are the applied axial load and resulting axial deformation. When loading biaxially and recording lateral expansion as well, a strong ‘Poisson ratio’ or lateral expansion may be experienced, which in the case of Type C (conjugate shear) will far exceed and isotropic elastic continuum limit (0.5) and even exceed 1.0 due to the combined effects of shearing and dilation.
shearing to maintenance of productivity. However this process, which was also illustrated in Figure 5, requires some shrinkage of the matrix to ‘make space’ for the shearing, if one-dimensional compaction is the average boundary condition, such as in the case of a compacting reservoir. Porous deformable chalk and weaker limestones would be suitable candidates.

Coupled shear flow tests (CSFT) performed by Makurat on Ekofisk chalk using either equilibrated sea water or Ekofisk oil, confirmed the maintenance of permeability due to shear, even under high effective stress levels. Some of the associated testing and modelling performed at NGI for the NPD (Norwegian Petroleum Directorate) was described in Barton et al. (1986, 1988). The CSFT method was described in detail in Makurat et al. (1990). Such coupled-process tests seem to be quite rare in the petroleum industry, although there are some notable PhD studies of shear-flow coupling in recent years potentially related with nuclear waste disposal scenarios.

A decade after these Norwegian coupled shear-flow studies for Ekofisk, the Stanford University group of Zoback and co-workers found convincing evidence of the importance of (interpreted) shear stress on whether fractures in crystalline rocks were conducting or not conducting, based on deep well-log analysis. The work of C. Barton et al. (1995) and Zoback and Townend (2001) is important confirmatory evidence of the conductivity-related value of shear stress. Joint- or fracture-shearing seems to be an essential component of successful gas-shale stimulation, but can be problematic for geothermal reservoir operation, as it was in the case of the Cornwall ‘hot dry rock’ project, if ‘fluid-capture-due-to-shear’ occurs, as sketched in Figure 5.

The importance of shear stiffness and associated block-size scale effects

Figure 10, from Barton (1982), demonstrates something which has potential application to understanding the process of gas-shale stimulation: namely the quite low shear stiffness which is also related with block size. To be strictly correct this should be termed the peak shear stiffness, which is shown defined in Figure 10, left-inset.

Both diagrams in Figure 10 show the effect of block size and normal stress on the resulting peak shear stiffness. The diagram on the left shows experimental data assembled by Barton (1982). As detailed in the figure caption, there is a beneficial double scale effect, meaning that in situ (larger blocks) (of e.g. gas-shale) will have significantly lower stiffness than laboratory (core-size) samples. The diagram on the right shows calculated peak shear stiffnesses for two different sets of input data. The lower example has JRC_n, JCS, and φ_r values that are more in line with gas-shales, but likely to be on the high side.

Shear stiffness in situ is significantly lower than generally listed in software company instruction ‘manuals’ as the double scale effect seems not to be widely appreciated, and shear stiffness is seldom or never mentioned when assessing the likely mobilization of friction in ‘critically stressed’ reservoirs. The reality is that friction starts to be mobilized at small strain, followed by the gradual mobilization of roughness towards peak shear strength. The use of JRC_n (mobilized) and other parameters that are also variables, gives greater understanding than using simple friction coefficient mobilization, which refers only to a single (peak) resistance. The key question is what level of effective normal
This appears to be an important message, and may result in changes to the stimulation techniques, due to shale fabric and stiffness changes, resulting in local rock stress changes. The word ‘shales’ is used mostly as a particle-size indicator. The shales may also be very fine-grained siltstones as indeed suspected in various locations around Kimmeridge Bay.

Examples of application of JRC and JCS

Brief examples of the application of JRC and JCS in petroleum engineering and in tunnelling are finally shown in order to demonstrate some additional features of non-linear coupled BB (UDEC-BB) modelling of rock mechanics (and potential reservoir) processes. Figure 12 illustrates one-dimensional stress is likely to be operating in a frac-stimulated gas-shale in the ‘surrounding’ volume, where fracture shearing is occurring? In addition, what is the typical block size for scaling purposes?

As an ‘anchor’ for such deliberations we may refer to the four photographs of interbedded shales and siltstones, from the unique North Sea source-rock location in Dorset in southern England: at Kimmeridge Bay. Here we can image the branching that is likely to occur when frac-stimulating gas-shale, as for instance described by Fisher and Warpinski (2011) and King (2010).

The examples of interbedded shales and siltstones in Figure 11 may have some possible relevance to the appearance of gas-shales in general because in the words of a gas-shale fracking reviewer like King (2010): ‘no two shales are alike’. This appears to be an important message, and may result in changes to the stimulation techniques, due to shale fabric and stiffness changes, resulting in local rock stress changes. The word ‘shales’ is used mostly as a particle-size indicator. The shales may also be very fine-grained siltstones as indeed suspected in various locations around Kimmeridge Bay.
Compaction modelling of a representative 1 x 1 m ‘window’ of porous-and-jointed parts of the 14 x 9 x 0.3 km Ekofisk reservoir in the North Sea. An assumed 20 MPa reduction in oil pressure causes sufficient effective stress increase to cause compaction of the non-linear matrix, such that space is made for (anisotropic) joint shearing (shown by ‘flags’), despite the one-dimensional restraint on compaction.

As a matter of ‘proof of shear’, slickensided polished joints were reportedly recognized by Phillips geologists (pers. comm. Helen Farrel), in recovered core from the mid-eighties Ekofisk water-flooding project. Such features had not been seen during exploration 15 years before. As also with gas-shale, some shearing is the key to enough dilation and the development and maintenance of some measure of permeability and therefore gas drainage.

The coupling of the desirable shear-dilation-permeability and the undesirable normal closure-permeability, and the differentiation of hydraulic ($e$) and physical ($E$) joint or fracture apertures, as occurring ‘in deep back-ground’ in the above numerical modelling, has been part of rock mechanics modelling since 1985, and it would seem to be applicable in the petroleum industry, if not already used, especially with the recognition of so much ‘remaining’ production (perhaps > 60%) from fractured and unconventional (shale) reservoirs.

Figure 13 is an example of coupled UDEC-BB modelling from a different field than reservoirs. Nevertheless, it is a convenient way to demonstrate the depth or stress dependence of the two apertures $E$ (physical: left diagram) and $e$ (theoretical smooth-wall hydraulic: right diagram). Note that shearing-induced dilation or tensile opening can open the physical aperture sufficiently for $e$ and $E$ to be equal. This usually occurs when apertures reach approximately 0.5 to 1.0 mm, as can be seen in Figure 14, for the case of the larger (routher-walled) JRC$_{mobilized}$ values.

Conversion between physical and hydraulic apertures was a subject that pre-occupied the author a long time ago. In Barton (1972) a graph was shown in which recent tests conducted at the University in Trondheim were interpreted such that the ratio of $E/e$ could be expressed as a function of the hydraulic aperture. Later, in Barton (1982) more data was collected and interpreted showing that the small-scale roughness JRC (strictly JRC$_s$) had an important role to play in the ratio $E/e$. This is logical as JRC can be estimated from the ratio of roughness amplitude ($a$) and length of profile ($L$), therefore being equivalent to relative roughness in hydraulics, though at much larger scale.

Figure 14 (left) shows the empirical model for converting between $e$ and $E$ in the case of normal closure. Successively, more experimental data is given in Barton (1982), Barton et al. (1985) and Barton and Quadros (1997). In the case of shearing with possible gouge production (Figure 14 right), the $E/e$ data inclines from left down to the right, crossing the curves on the left. In situ block test data had shown this in 1985, and a formal improvement in the model converting $E$ to $e$ and vice versa, was published by Olsson and Barton (2001), following PhD studies of Olsson externally-examined by the writer. The term JRC$_{mobilized}$ was used in the new conversion of $e$ to $E$ for the case of shear. So we have the two forms:

For normal closure: $e = E^2/JRC_s^{2.5}$

For shear (and possible gouge): $e = E^{1/2} JRC_{mobilized}$

**Coupled shear-displacement dependent behavior**

The dimensionless model for JRC$_{mobilized}$ shown in Figure 15 is a useful device for understanding the consequences of shear displacement: namely the mobilization of friction, then mobilization of roughness and eventual degradation of roughness post peak. This empirical model, designed to

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**Figure 13** UDEC-BB model of twin tunnels in limestones and shales, from Makurat and Barton (1988). This non-reservoir case is presented here in order to illustrate the two modelled apertures and also to demonstrate that they are both depth or stress dependent. Of course both apertures will be extremely small (a few microns size) at kilometres depth in gas-shales, until shearing and (slight but sufficient) dilation occurs. Fracture aperture $E$ (physical) and $e$ (theoretical) are both tracked in the non-linear Barton-Bandis model, based on the JRC$_s$ dependent conversion from $E$ to $e$ shown in Figure 14. Note: each line thickness in Figure 13 = 20 micron. Permeability, or more correctly joint- or fracture-conductivity is determined by the smaller hydraulic aperture ($e$) (with $k = e/12$), unless the matrix is significantly permeable as well. In the different context of reservoirs, petroleum reserves lie in the matrix pores and in the larger physical joint- or fracture-apertures ($E$), especially in the case of fractured basement reservoirs in granites, which have high fracture porosity and low matrix porosity. The apertures which can be back-calculated from flow tests or from interpretation of production are the usually much smaller hydraulic apertures ($e$), so reserves in basement fractured reservoirs need to be greatly upgraded if the $E$/$e$ mismatch has not been allowed for.
match measured reality, is of course more sophisticated than just the discussion of ‘friction coefficient’ (mobilized or available) in so-called critically stressed reservoirs, following the ‘Stanford’ model, which is seemingly referred to and used by most petroleum companies.

More important than its apparent lack of application outside rock mechanics, is that JRC mob provides the ability to generate shear-stress-displacement and dilation-displacement, and therefore also permeability-displacement curves for any desired sets of BB input data. The dimensionless model was developed in Barton (1982) as a means of representing shear tests results. Direct shear test results are also apparently rare in petroleum geomechanics. This non-linear part of rock mechanics practice does not appear to be part of the already complicated petroleum geomechanics, although thousands of additional citations to such methods in the last five to six years perhaps suggests that changes are occurring.

The shear-displacement and dilation-displacement diagrams shown in Figure 16 indicate approximate estimates of the displacement needed to reach peak shear strength ($\delta_{\text{peak}}$), which is an important component of the shear stiffness estimate. It is found that peak strength may be reached after a shear displacement of about 1% of the sample length in the case of small (100 mm long) laboratory samples, but this percentage reduces with larger block sizes.

A large body of such data (some 600 direct shear tests on widely different sizes of samples) was collected in Barton (1982). Unfortunately there is a wide scatter in the experimental results, so there will always remain some uncertainty in the shear stiffnesses and in the onset of dilation, and in the position of the maximum dilation angle, which logically occurs simultaneously with peak shear strength. Note from Figure 14, that when apertures reach about 0.5 to 1 mm (due to shear-induced dilation), there will be little difference between e and E. However, in lower modulus gas-shales, there will likely be an earlier onset of gouge-with-shear production, which is certainly one of the reasons that higher modulus (brittle) shales are favoured (e.g. King, 2010).

Figure 14 Left: This E/e conversion (from Barton, 1982) applies to normal closure. When shear (and possible gouge production) is involved, a modified form of E/e conversion is used, using the 1982 JRC mobilized concept. This is shown on the right, from Olsson and Barton (2001). In practice the ‘JRC-curves’ are steeper and incline upwards to the right as in the original model. JRC mobilized tracks the mobilization of roughness pre-peak and the degradation of roughness post-peak. It is shown in Figure 15. In situ block test data from Barton et al. (1985), and laboratory shear-flow data from Olsson (Olsson and Barton, 2001) gave evidence for the need for this adjustment for shear, with possible permeability losses due to gouge. Note that the permeability eventually increases most in the direction perpendicular to the shearing direction, as suggested and demonstrated by Gentier (1987) in the late Eighties.

Figure 15 An essential part of coupled process modelling involving joint or fracture shearing is the ability to track shear deformation. The JRC mobilized dimensionless model shown in this figure was developed by Barton (1982). This version is from Barton (2006, Chapter 16 concerning shearing processes in rock mechanics). When one conducts direct shear tests on different joint or fracture samples at widely different normal stress levels, a series of widely differing shear stress-displacement (and dilation-displacement) curves are obtained. The dimensionless quantities (JRC mob and $\delta_{\text{peak}}$) represented in the ‘universal’ diagram shown here, has the effect of consolidating all experimental shear test results into one narrow band with the approximate shape shown here. We can therefore use this single simple device to generate widely different shear stress-displacement and dilation-displacement curves for any desired input data (including variable block size and variable effective normal stress, as shown in Figure 16. The ‘look-up’ table of values (see inset) is of course smoothed in the UDEC-BB sub-routine.
Figure 16 An important and favourable feature of stimulated production of gas-shale is that the shear strength of rock joints or fractures is block-size dependent, as shown in this hand-calculated demonstration from Barton (1982). Shearing may be 'easier' than expected. Note that the space created by shearing-induced dilation will be compromised at high effective normal stress, due to reduced dilation and possible/probable gouge production. It will be noticed that a dilation ‘delay’ is involved, which may be something to consider when stimulating gas-shales. In other words a significant fracking and propping is needed in the ‘central’ ellipsoidal volume to push the surrounding shale beyond the ‘dilation delay’. The latter will be more marked when block size is larger, as can be seen in the right-hand diagram. However, only a very few millimetres of shear is needed to greatly enhance the potential permeability. Even with gouge production, the coupled shear-flow tests of Olsson showed one and a half orders of magnitude of conductivity increase with only 4 to 5 mm of shear deformation. The models shown in Figure 17 are without any gouge correction, but the E/e conversion already corrects for the tortuosity/out-of-plane effects of rock-to-rock contact across the stressed joints or fractures.

Figure 17 It is appropriate to finish this introduction to non-linear rock mechanics, by assembling an early series of BB coupled-behaviour models generated by Bakhtar on an HP programmable calculator in 1983, while he and the writer were working in TerraTek (now Schlumberger) in Salt Lake City. This was part of a two-volume nuclear waste related study for AECL/CANMET in Canada. These figures were also presented in Barton et al.(1985). The sets of three curves on the left and right are designed to demonstrate the likely effect of block size (on the left), and the effect of effective normal stress (on the right), at constant 300 mm block length (on the right), a possibly shale-like block size. Note that the third or lowest set of curves in each case are based on the empirical conversion from physical aperture E to hydraulic aperture e, with an assumed starting point, for these examples, of \( e_0 = 25 \mu m \). In practice, due to the JRC\(_{residual}\) mechanism and possible gouge production, the increased conductivity with shear deformation may be less than graphed here, but is likely to remain impressive at least while the shale (or carbonate/chalk) remains brittle or nearly brittle.
The more planar jointing or natural fracturing of gas-shales which can be seen in occasional photographs of exposures of shales in the literature, a planarity also represented in Kimmeridge Bay shales in Figure 11, suggests that a significantly weaker scale effect will be seen in the case of gas-shales than the ‘strong-scale-effect’ examples represented in Figure 16.

Nevertheless, the assumed peripheral shearing and source of microseismic activity registered in frac-stimulated gas-shales, will be occurring ‘more easily’ than might be suspected if ‘only’ lab-scale coupled shear flow tests (CSFT) have been performed by the industry since the ‘discovery’ of producible/frac-able gas-shales nearly a decade ago.

The same conclusion can be levelled at the 1985 CSFT tests performed on conjugate Ekofisk joints. Production/compaction induced shearing occurs more easily than in laboratory tests because of the larger in situ block sizes. This is why it is useful to have discrete fracture models like UDEC-BB for investigating possible mechanisms at convenient scale (as in Figure 12), before attempting to up-scale the phenomena and represent likely trends with time in reservoir models.

As will probably have been understood from this introduction to non-linear (non Mohr-Coulomb) rock mechanics, a strong contribution to an understanding of the effect of time and reduced pressure should be possible, if some of the methods shown are adopted in petroleum geomechanics. The possible need for water flooding earlier rather than later might also be appreciated, after rejecting linear strength envelopes, which cannot apply to the petroleum industry’s usually large changes of effective stress.

Conclusions
Non-linear shear strength, as opposed to inaccurate Mohr-Coulomb linearity, plus nearly critical-state (horizontal strength envelope) conditions may be experienced by weaker reservoir rocks, during their declining pore pressure history.

Non-linear shear strength, dilation and permeability coupling of joints and fractures is likely to give a more realistic prediction of NFR 4D potential, and of gas-shale stimulation performance, than anything linked to linear Mohr-Coulomb. The over-frequent use of the latter needs a serious review.

Geomechanics in general and fracture characterization in particular, as apparently practised in the petroleum industry, could usefully be extended into something more resembling rock mechanics practices, where deformation, dilation and coupling with permeability have been of fundamental interest for many decades. This would also now be expected in an industry apparently so dependent on future production from mostly fractured reservoirs.

Inevitably there may be impressive ongoing rock mechanics modelling of the non-linear coupled processes discussed in this paper. This may be occurring in some large oil and service companies, or in sponsored university departments. The fact that such work is not readily published or described in workshops means that the present author sympathizes beforehand with those who cannot easily respond to say ‘we have also been doing that’.

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